# Optimal Stopping, Sequential Methods, and Related Topics

April 28-29, 2011

Organisation Hans Rudolf Lerche Dominik Stich

Universität Freiburg Mathematisches Institut Abteilung für Mathematische Stochastik





Albert-Ludwigs-Universität Freiburg Mathematical Institute Department for Mathematical Stochastics

Hans Rudolf Lerche Dominik Stich

## **Optimal Stopping, Sequential Methods, and Related Topics**

Thursday, April 28, 2011	
9:00- 9:15	— Welcome —
9:15–10:00	<b>Paavo Salminen</b> , Åbo Akademi University (Finland) <i>Optimal stopping, Appell polynomials and Wiener–Hopf factorization</i>
10:00-10:40	<b>Mihail Zervos</b> , London School of Economics and Political Science (United Kingdom) On the optimal stopping of a one-dimensional Itô diffusion
10:40-11:00	– Coffee Break — in room 331 —
11:00-11:40	Frank Riedel, University of Bielefeld Optimal stopping under Knightian uncertainty
11:40-12:10	<b>Yan Dolinsky</b> , ETH Zurich (Switzerland) Application of strong approximation theorems to optimal stopping
12:10-12:40	<b>Elena Boguslavskaya</b> , London School of Economics and Political Science (United Kingdom) Solving optimal stopping problems with Appell functions
12:40-14:00	— Lunch —
14:00-14:40	<b>Albrecht Irle</b> , University of Kiel American options with guarantee for diffusions and Lévy processes
14:40-15:10	<b>Sören Christensen</b> , University of Kiel A method for pricing American options using semi-infinite linear programming
15:10-15:20	— Short Break —
15:20–15:50	<b>Christian-Oliver Ewald</b> , University of Sydney (Australia) Asymptotic solutions for real option under stochastic volatility
15:50–16:20	<b>Daniel Jones</b> , Technical University Darmstadt Optimal exercising of American options in discrete time via forecasting of sta- tionary and ergodic time series
16:20-17:15	— Coffee Break — in room 331 — Walk of 5 minutes to Albertstr. 19
17:15–18:00	<b>David Siegmund</b> , Stanford University (USA) <i>Multiple comparisons in searching for local signals</i> !!! Talk is at FRIAS, Albertstr. 19 !!!

19:30 Joint Dinner at Dattler (only with special registration)



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## Friday, April 29, 2011

9:00- 9:45	<b>Albert N. Shiryaev</b> , Steklov Institute Moscow (Russia) Local time approach to the Bayesian problems of testing two and three hypo- theses for Brownian motion with drift
9:45–10:30	<b>Goran Peskir</b> , University of Manchester (United Kingdom) <i>Optimal detection of a hidden target</i>
10:30-10:50	— Coffee Break — in room 331 —
10:50-11:25	<b>Ernst Presman</b> , Central Economics and Mathematical Institute, Moscow (Russia)
	Construction of the value function by a modification of the payoff function in the optimal stopping of one-dimensional diffusion with finite number of peculiarities
11:25–11:55	<b>Pavel V. Gapeev</b> , London School of Economics and Political Science (United Kingdom)
	About two-dimensional Bayesian disorder problems
11:55–12:25	<b>Claudia Kirch</b> , Karlsruhe Institute of Technology On the bootstrap for sequential change-point tests
12:25-12:45	<b>David Siegmund</b> , Stanford University (USA) <i>Multiple comparisons in searching for local signals</i> , Part II
12:45-14:00	— Lunch —
14:00-14:35	<b>Chris A. J. Klaassen</b> , University of Amsterdam (The Netherlands) <i>Bonus – Malus in acceptance sampling</i>
14:35–15:10	<b>A. V. Gnedin</b> , Utrecht University (The Netherlands) Sequential selection of random chains in self-similar posets
15:10–15:40	<b>Mikhail Urusov</b> , Universität Ulm On the martingale property of exponential local martingales
15:40-16:00	— Coffee Break — in room 331 —
16:00–16:30	<b>Aleksandar Mijatović</b> , University of Warwick (United Kingdom) On the drawdown of completely asymmetric Lévy processes
16:30-17:00	Jan-Hendrik Steg, Univerität Bielefeld Singular control games
17:00–17:30	<b>Vladimir Mazalov</b> , Institute of Applied Mathematical Research, Karelia (Russia) <i>On CUSUM procedures for exponential distribution</i>
17:30–18:15	<b>Moshe Pollak</b> , Hebrew University of Jerusalem (Israel) Selecting "good" sets

## Abstracts

**Elena Boguslavskaya**, London School of Economics and Political Science (United Kingdom)

## Solving optimal stopping problems with Appell functions

Recently, there were series of papers by Novikov and Shiryaev, Kyprianou, Levendorski, Mordecki, Salminen, Surya, where the solution of optimal stopping problems for Lévy processes and random walks were found in terms of the maximum/minimum of the process. We continue in the same spirit. In the present paper we propose the method similar to the one presented in the papers by Surya, and Novikov and Shiryaev. To find the optimal stopping boundary explicitly, we construct the Appell function, and find the desired boundary as a root of this function.

The so-called Appell function happens to be an Esscher–Laplace transform, and we also show how the Appell function can be represented as series in Appell polynomials. As Appell polynomials are complete Bell polynomials in cumulants, it offers an easy way to calculate coefficients of Appell polynomials as "inverse moments". Moreover, we show how the above mentioned representation leads naturally to the martingale property of Appell polynomials generated by Lévy processes.

To conclude, we provide several examples, including the cases when the underlying processes are Brownian motion, Poisson process, spectrally negative or spectrally positive processes.

## Sören Christensen, University of Kiel

## A method for pricing American options using semi-infinite linear programming

A new approach for the numerical pricing of American options is introduced. The main idea is to choose a finite number of suitable excessive functions (randomly) and to find the smallest majorant of the gain function in the span of these functions. The resulting problem is a linear semi-infinite programming problem, that can be solved using standard algorithms. This leads to good upper bounds for the original problem. For our algorithms no discretization of space and time and no simulation is necessary. Furthermore it is applicable even for high-dimensional problems. The algorithm provides an approximation of the value not only for one starting point, but for the complete value function on the continuation set, so that the optimal exercise region and e.g. the Greeks can be calculated. We apply the algorithm to (one- and) multidimensional diffusions and to Lévy processes, and show it to be fast and accurate.

## Yan Dolinsky, ETH Zurich (Switzerland)

## Application of strong approximation theorems to optimal stopping

We derive error estimates for discrete approximations of optimal stopping values which are defined on a *d*-dimensional Brownian probability space. We consider a general path-dependent payoffs with some regularity properties. Our main tool is based on strong approximation theorems for i.i.d. random vectors, which were obtained by Sakhanenko (2002). We also show how to use this method to approximate American options values in Jump-diffusion models.

## Christian-Oliver Ewald, University of Sydney (Australia)

## Asymptotic solutions for real options under stochastic volatility

We derive asymptotic solutions for real options under stochastic volatility. We consider modelling the project's value with processes that resemble both a geometric Brownian motion and a geometric mean reverting process, but their variances being modelled using a CIR process. We further derive the relationship between the threshold (the point at which to undertake a project or not) and the parameters used in the stochastic volatility model. We compare these results to the classical case of non-stochastic volatility.

## Pavel V. Gapeev, London School of Economics and Political Science (United Kingdom) About two-dimensional Bayesian disorder problems

We study the Bayesian disorder detection problems in a model in which two observable constantly correlated Wiener processes change their drift rates at some independent exponential times which are inaccessible for observation. The initial problems are reduced to optimal stopping problems for multi-dimensional continuous Markov processes called sufficient statistics. We derive stochastic differential equations for the sufficient statistics in the disorder problem for the case of linear and exponential delay penalty costs. The optimal stopping times of alarm are sought as the first times at which sufficient statistics processes exit certain regions restricted by non-constant boundaries. By means of the change-of-variable formula with local time on surfaces, it is shown that the optimal stopping boundaries can be uniquely characterized as solutions of the associated free-boundary problems. We also derive several explicit estimates for the initial Bayesian risk functions and the optimal stopping boundaries.

#### **A. V. Gnedin**, Utrecht University (The Netherlands)

#### Sequential selection of random chains in self-similar posets

We consider sampling from a space S endowed with a partial order and a probability measure. All upper cones in S are assumed similar up to a scaling factor. The standard example is a d-dimensional cube with the uniform distribution. The problem is to select, in nonanticipating fashion, an increasing subsequence of independent n-sample from S. We compare the lengths of selected sequences under a greedy policy (selecting the 'chain records') and a policy which may reject some of the options that satisfy the order constraint.

#### Albrecht Irle, University of Kiel

### American options with guarantee for diffusions and Lévy processes

We introduce American options with guarantee which provide a safety belt against a substantial loss of fortune. So we consider American options with payoff  $g(X_{\tau}) \lor h(X_0)$ ; here  $(X_t)_{t\geq 0}$  is the stock price process, g, h are increasing functions,  $\tau$  is the (random) time to exercise the option and  $\lor$  denotes the maximum. For pricing we are faced with the optimal stopping problem

$$(x) = \sup_{\tau \in \mathcal{T}} E_x(e^{-r\tau}(g(X_\tau) \lor h(x))),$$

where r is the discounting factor and we treat the perpetual case, i.e. infinite time horizon. We consider this class of problems for diffusion processes and Lévy processes. The general theory of optimal stopping for one-dimensional Markov processes does not apply immediately since the payoff function depends on the starting point. One possibility is to embed the problem into a two-dimensional problem. From this point of view, structural results can be obtained but explicit calculations seem to be hard to handle.

In many situations optimal stopping rules for problems with discounting have the form  $\tau^* = \inf\{t \ge 0 : X_t \ge a\}$  for some a. This is different in our case. Assume that you want to stop at a reasonable high level a and you are far below it. Then you have the option to stop and accept the guarantee. This intuition is correct and we arrive at the situation, uncommon in American option pricing, that our stopping rules are two-sided. By using the theory of minimal r-harmonic functions we prove the following result for diffusions under minimal conditions:

For each x with h(x) > 0 there exist constants  $a_x \le x \le b_x$  such that for  $\tau_x = \inf\{t \ge 0 : X_t = a_x \text{ or } X_t = b_x\}$  we have

$$v(x) = E_x(e^{-r\tau_x}(g(X_{\tau_x}) \lor h(x))).$$

The explicit computation of the optimal boundaries and the function v for all starting points x leads to a system of two coupled first-order ODEs. We derive these equations and treat various examples. These results are then extended to Lévy processes with particular attention to the spectrally negative case.

Joint work with Sören Christensen, University of Kiel.

#### Daniel Jones, Technical University Darmstadt

# Optimal exercising of American options in discrete time via forecasting of stationary and ergodic time series

The problem of exercising an American option in discrete time in an optimal way is considered, i.e. maximization of the expected discounted payoff. The algorithm proposed uses techniques of forecasting of time series and is completely nonparametric in the sense that it is solely based on observations of the underlying asset. It is shown that the expected payoff of the corresponding stopping rule converges to the optimal value whenever the returns of the underlying asset are stationary and ergodic.

Joint work with Michael Kohler, Technical University of Darmstadt.

### **Claudia Kirch**, Karlsruhe Institute of Technology (KIT)

#### On the bootstrap for sequential change-point tests

Change-point analysis deals with the detection of structural breaks in time series. In a sequential setting the data arrive one by one and after each new observation one checks whether a change has occurred. This approach is natural for many applications such as monitoring intensive care patients, financial time series or climate data. Critical values are based on distributional asymptotics but do not work well if there are only few historical observations (control data). Bootstrapping methods have widely been used in a non-sequential setting, however, there is hardly any literature on variations or the validity of the bootstrap in a sequential setup. In this talk we will investigate these questions using a simple mean change model as well as a linear regression model. In particular we consider how to make use of the new incoming observations which can improve the bootstrap estimate. From a practical point of view a repeated bootstrap after each observation has high computational costs so variations are of interest. From a theoretical point of view the critical values change with each incoming observation, so the question is whether this procedure remains consistent.

This is joint work with Marie Hušková (Prague).

#### References

- [1] Hušková, M., Kirch, C.: Bootstrapping sequential change-point tests for linear regression. *Metrika*, 2011. To appear.
- [2] Kirch, C.: Bootstrapping sequential change-point tests. Seq. Anal., 27:330-349, 2008.

#### Chris A. J. Klaassen, University of Amsterdam (The Netherlands)

#### Bonus – Malus in acceptance sampling

In acceptance sampling we consider a continuing sequence of lots of items of possibly varying quality, where the number of items in the lots may vary as well. From a lot of size N a sample of size n is taken, the items in the sample are inspected, and based on the results the lot is accepted or rejected. As performance criterion we define the expected outgoing quality at rejection. A sampling and acceptance strategy is presented for which the expected outgoing quality at rejection satisfies a given bound.

More precisely, in acceptance sampling on attributes we reject if the sample contains a defective item, we denote credit by K, and we define it as the number of items accepted since the last lot has been rejected. Then the sample size

$$n = \left\lceil \frac{N}{(K+N)\pi + 1} \right\rceil$$

guarantees an expected outgoing quality at rejection of  $\pi$ .

So, the more credit, the less inspection. In this way a producer who has built up a lot of credit, gets a bonus, namely less inspection costs.

## Vladimir Mazalov, Institute of Applied Mathematical Research, Karelia (Russia) On CUSUM procedures for exponential distribution

We obtain a closed form representation for the expectation of the first passage time, known also as the Avarage Run Length for the CUSUM process, in the case of Exponential type distribution. To evaluate the ARL for a CUSUM procedure we use the integral equation approach.

## Aleksandar Mijatović, University of Warwick (United Kingdom)

## On the drawdown of completely asymmetric Lévy processes

The drawdown process  $Y = \overline{X} - X$  of a completely asymmetric Lévy process X is given by X reflected at its running supremum  $\overline{X}$ . In this paper we explicitly express the law of the sextuple  $(\tau_a, \overline{G}_{\tau_a}, \underline{X}_{\tau_a}, \overline{X}_{\tau_a}, Y_{\tau_a-}, Y_{\tau_a} - a)$  in terms of the scale function and the Lévy measure of X, where  $\tau_a$  denotes the first-passage time of Y over the level a > 0,  $\overline{G}_{\tau_a}$  is the time of the last supremum of X prior to  $\tau_a$  and  $\underline{X}$  is the running infimum of X. We also explicitly identify the distribution of the drawup  $\widehat{Y}_{\tau_a}$  at the moment  $\tau_a$ , where  $\widehat{Y} = X - \underline{X}$ , and derive the probability of a large drawdown preceding a small rally. Explicit formulae, in terms of Mittag-Leffler functions, are obtained when the log-stock is modelled by a spectrally one-sided  $\alpha$ -stable process.

This is joint work with Martijn R. Pistorius, Imperial College London.

## Goran Peskir, University of Manchester (United Kingdom) Optimal detection of a hidden target

We show that in the absence of any information about the 'hidden' target in terms of the observed sample path, and irrespectively of the distribution law of the observed process, the 'median' rule is optimal in both the space domain and the time domain. While the fact that the median rule minimises the spatial expectation can be seen as a direct extension of the well-known median characterisation dating back to R. J. Boscovich, the fact that this also holds for the temporal expectation seems to have stayed unnoticed until now. We will present a review of the recent results building on this observation.

The talk is dedicated to the tercentenary of the birth of R. J. Boscovich (1711–1789).

## Moshe Pollak, Hebrew University of Jerusalem (Israel) Selecting "good" sets

Consider having to collect a group of "goodïtems (people, investments, whatever). Items come up for scrutiny one at a time, independently, and it is necessary to decide on the spot whether or not to add an item to the collection, without an option to change one's mind. Sometimes nothing is known in advance about the population from which the collection is to be picked out; at other times everything is known. Being hasty in accepting an item will lead to a poor overall quality of the accumulated set; being too timid will cause the set to grow very slowly. How may one go about this?

In this talk, several scenarios will be regarded and procedures will be proposed.

#### Ernst Presman, Central Economics and Mathematical Institute, Moscow (Russia)

## Construction of the value function by a modification of the payoff function in the optimal stopping of one-dimensional diffusion with finite number of peculiarities

A problem of optimal stopping of a one-dimensional time-homogeneous diffusion with the infinite horizon is considered. The diffusion takes values in a finite or infinite interval ]a, b[. The points a and b may be either natural or absorbing or reflecting. The diffusion is regular with the exception of a finite (may be empty) set of points  $A^0$ , where a partial reflection take place. A discounting and a cost of observation are allowed. Both can depend on the state of the diffusion. The payoff function g(z) is bounded on any interval [c, d], where a < c < d < b, and twice differentiable with the exception of a finite (may be empty) set of points  $A^1$ , where the functions g(z) and q'(z) may have a discontinuities of the first kind. Let L be an infinitesimal generator of diffusion which includes the members corresponding to the discounting and the cost of observation. We assume that the set  $\{z: Lq(z) > 0\}$  consists of a finite number of intervals. The set of ends of these intervals we denote by  $A^2$ . For such problem we propose a procedure of constructing the value function in a finite number of steps. In the process of constructing we do not need to guess about the structure of the stopping set and we do not need to use any verification theorem. The procedure is based on the fact that in the neighborhood of peculiarities one can modify the payoff function. Under peculiarities we mean the points from  $A^0 \bigcup A^1 \bigcup A^2$  and also the points a and b if they are reflecting. The idea of the procedure is connected with the Sonin's elimination algorithm (see [1]). A preliminary result was published in [2].

The work was supported by RFBR grant 10-01-00767-a.

#### References

- [1] I. M. Sonin, The elimination algorithm for the problem of optimal stopping. *Math. Meth.* of Oper. Res. 49, 111–123 (1999).
- [2] E. L. Presman, Solution of optimal stopping problem based on a modification of payoff function, Musela Festschrift, Springer Verlag, 2011.

#### Frank Riedel, University of Bielefeld

### Optimal stopping under Knightian uncertainty

We consider the problem of stopping optimally a sequence of random payoffs. Such optimal stopping problems arise frequently in economics, finance, operations research, and (sequential) statistics. In recent years, the problem of model uncertainty has drawn a lot of attention in economics and finance. One way to model such uncertainty is to use a whole class of priors combined with a worst-case approach. We develop a theory of optimal stopping under such model uncertainty in discrete as well as continuous time and apply it to several examples from Mathematical Finance including shout and barrier options.

#### Paavo Salminen, Åbo Akademi University (Finland)

## Optimal stopping, Appell polynomials and Wiener–Hopf factorization

In this talk the optimal stopping problem for Lévy processes for the reward function  $(x^+)^n$  studied by Novikov and Shiryayev is revisited. In particular, we are interested in finding the representing measure of the value function. It is seen that that this can be expressed in terms of the Appell polynomials. An important tool in our approach and computations is the Wiener–Hopf factorization.

#### Albert N. Shiryaev, Steklov Institute Moscow (Russia)

# Local time approach to the Bayesian problems of testing two and three hypotheses for Brownian motion with drift

We observe a Brownian motion  $X = (X_t)_{t \ge 0}$  with drift  $\mu$ ,

$$X_t = \mu t + B_t,$$

where  $\mu = 0$  or 1 (with probability 1/2, for the case of two hypotheses) and  $\mu = -1, 0, 1$  (with probability 1/3, for the case of three hypotheses). For Bayesian risks

$$R_{\delta} = \mathsf{E}(c\tau + W(\mu, d)), \qquad \delta = (\tau, d),$$

with  $W(\mu^i, d^i) = 0$  and  $W(\mu^i, d^j) = 1$ ,  $i \neq j$ , we show that in case of two hypotheses the following representations hold:

$$R_{\delta} = \mathsf{E}\Big[c\tau - \frac{1}{4}L_{\tau}^{+}\Big],\tag{1}$$

where  $L_t^+$  is a local time on the ray x = t/2 of the process  $X = (X_t)_{t \ge 0}$  with

$$dX_t = \frac{\exp\{X_t - t/2\}}{1 + \exp\{X_t - t/2\}} dt + d\overline{B}_t$$

(innovation representation) and for the case of three hypotheses

$$R_{\delta} = \mathsf{E}\left[c\tau - \frac{1}{2}\int_0^{\tau} \frac{dL_s^{\pm}}{2 + e^{-s}}\right],\tag{2}$$

where  $L_t^{\pm}$  is a local time on the rays  $x=\pm t/2$  of the process  $X=(X_t)_{t\geq 0}$  with

$$dX_t = \frac{\exp\{X_t - t/2\} - \exp\{-X_t - t/2\}}{1 + \exp\{X_t - t/2\} + \exp\{-X_t - t/2\}} dt + d\overline{B}_t$$

(innovation representation), where  $\overline{B}$  is a Brownian motion.

We use representations (1) and (2) to analyze the structure of optimal stopping times for the cases of testing two or three hypotheses.

The talk is based on our joint paper with M. Zhitlukhin.

#### **David Siegmund**, Stanford University (United States)

#### Multiple comparisons in searching for local signals

Advances in one's ability to collect large amounts of data in biology/genetics and elsewhere lead to statistical problems that involve searching an indexed set of observations for signals that are manifested as "peaks" against a noisy background. I will describe a number of examples and discuss an approach to the problem of multiple comparisons, with applications to detection of copy number variations and paired end reads to detect insertions and deletions.

This is joint research with Benny Yakir and Nancy Zhang.

#### Jan-Hendrik Steg, Univerität Bielefeld

#### Singular control games

We present a duopoly model of strategic capital accumulation in continuous time with uncertainty, such that investment takes the form of singular control. Spot competition is of Cournot type. For this model there exists a parameterized and Pareto-rankable family of Markov perfect equiblibria in symmetric strategies, according to which implicit collusion induces positive option values. However, preemption can also eliminate any option value in a limiting case corresponding to Bertrand prices.

#### Mikhail Urusov, Universität Ulm

### On the martingale property of exponential local martingales

The stochastic exponential Z of a continuous local martingale M is itself a continuous local martingale. We give a necessary and sufficient condition for the process Z to be a true martingale and for the process Z to be a uniformly integrable martingale in the case where  $M_t = \int_0^t b(Y_u) dW_u$ , the process Y is a one-dimensional diffusion, and the process W is a Brownian motion. These conditions are deterministic and expressed only in terms of the function b and the drift and diffusion coefficients of Y. Such questions arise in stochastic analysis and its applications whenever one needs to perform a (locally) absolutely continuous measure change. For instance, one can characterize several notions of no-arbitrage and examine relations between them, or characterize one-dimensional diffusion models with bubbles using our results.

This is a joint work with Aleksandar Mijatović.

## Mihail Zervos, London School of Economics and Political Science (United Kingdom) On the optimal stopping of a one-dimensional Itô diffusion

We consider the one-dimensional Itô diffusion X that satisfies the stochastic differential equation

$$dX_t = b(X_t) dt + \sigma(X_t) dW_t$$
(1)

in the interior  $\operatorname{int} \mathcal{I} = ]\alpha, \beta[$  of a given interval  $\mathcal{I} \subseteq [-\infty, \infty]$ , where  $b, \sigma : \operatorname{int} \mathcal{I} \to \mathbb{R}$  are Borel-measurable functions and W is a standard one-dimensional Brownian motion. We allow for the endpoints  $\alpha$  and  $\beta$  to be inaccessible or absorbing. The objective of the discretionary problem that we study aims at maximising the performance index

$$\mathbb{E}_{x}\left[\exp\left(-\int_{0}^{\tau}r(X_{t})\,dt\right)f(X_{\tau})\mathbf{1}_{\{\tau<\infty\}}\right]$$
(2)

over all stopping times  $\tau$ , where the reward function  $f : \mathcal{I} \to \mathbb{R}_+$  and the discounting rate function  $r : \mathcal{I} \to \mathbb{R}_+$  are Borel-measurable. We derive a simple necessary and sufficient condition for the value function v of this problem to be real-valued. In this case, we show that v is the difference of two convex functions, and we prove that it satisfies the variational inequality

$$\max\left\{\frac{1}{2}\sigma^2 v'' + bv' - rv, \ \overline{f} - v\right\} = 0 \tag{3}$$

in the sense of distributions, where  $\overline{f}$  is the upper semicontinuous envelope of f. Conversely, we establish boundary conditions at the endpoint,  $\alpha$ ,  $\beta$  that a solution of (3) should satisfy to identify with the value function v. Furthermore, we recover several characterisations of the solution to the optimal stopping problem that have been studied in the literature, and we derive a generalisation of the so-called "principle of smooth fit" that can be used to obtain

explicit solutions in special cases of the general problem. Our analysis is based on a complete characterisation of differences of convex functions in terms of appropriate resolvents of the Itô diffusion X. In particular, we use this characterisation to show that a function  $F: \mathcal{I} \to \mathbb{R}_+$  is excessive if and only if it is the difference of two convex functions and  $\frac{1}{2}\sigma^2 F'' + bF' - rF$  is a positive measure.

This is joint work with Damien Lamberton.

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Mathematical Institute Department for Mathematical Stochastics Eckerstr. 1 79104 Freiburg Administration: Monika Hattenbach, Thomas Lais serketariat@stochastik.uni-freiburg.de Tel.: +49(0)761/203-5669/-5664 Fax: +49(0)761/203-5661 Organization: Prof. Dr. Hans Rudolf Lerche lerche@stochastik.uni-freiburg.de Tel.: +49(0)761/203-5662 Fax: +49(0)761/203-5661